Masterclass – **Theory of Corbelling**

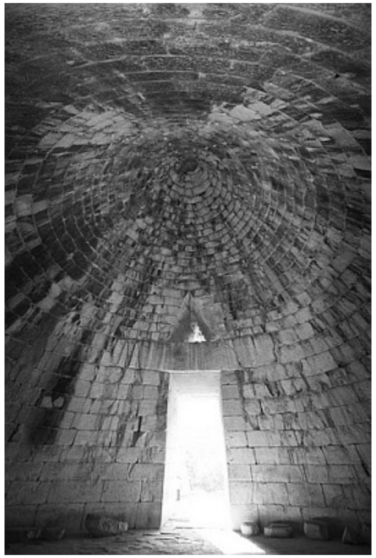
Not sure this should really be called a masterclass seeing as how I've never really carried out structural corbelling. Mind you I don't suppose it's ever stopped me spouting on authoritatively before, so why stop now? I have of course used the principle of cantilevering, have some experience of in effect corbelling the bottom and top of the vase whose construction was shown in "Stonechat 16", and have a smattering of an understanding of mathematics and structural physics so here goes nothing… and by the way having a pack of playing cards handy might prove useful.

According to OED a Corbel is *"a projection of stone, wood etc., jutting out from a wall to support weight"*, about.com:architecture defines it as *"an architectural bracket or block projecting from a wall and supporting (or appearing to support) a ceiling, beam, or shelf."* some definitions imply projections supporting arches etc. Basically in walling terms it is a projecting stone which supports another stone and the process of corbelling is in effect where one corbel sits on top of another, and another…

The technique dates well back into history with the roofs of many Neolithic tombs corbelled until the gap could be closed by slabs. The ancient Greeks used the method a Tirnys to create passageways and Mycenae where the Treasury of Atreus/Tomb of Agamemnon has a corbelled relieving arch over the lintelled entrance with the interior a spectacular corbelled dome

Treasury of Atreus, Mycenae, Greece © Sean Adcock

These triangles are often referred to as 'false arches', as unlike arches the structure is not self supporting, similarly a corbelled dome is frequently referred to as a 'false dome'. Professor Borut Juvanec provides a useful definition of the distinction in http://www.stoneshelter.org/stone/construction.htm. The difference between corbelling and cupolas [domes] is that in corbelling layers are horizontal… while in cupolas they perpendicularly follow the construction plane.



Inside Treasury of Atreus By kind permission Rob Brooks-Bilson, all rights reserved

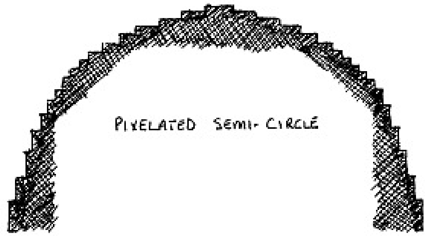
Interestingly at Tiryns there are semi-corbelled arches where the voussoirs (arch stones) are corbelled rather than radiating from a central point (and shaped to do so) but the top is closed with a keystone rather than the last two corbels meeting, (JE Gordon "Structures: Or Why Things Don't Fall Down" Penguin, 1978. p.187).

Now for some playing cards… Try poking one out from the edge of table until you find its point of balance. It should come as no surprise that this leaves the card half on and half off the table. Now try adding another on top of the first and try overhanging it a little further. It falls off. Now try taking 2 cards overlapping them half and half and try balancing the pair on the table. They should be stable if about ¼ of lower card is sticking out.

This is all about centres of gravity. In the first instance the centre of gravity of a single card, in the second where the centre of gravity is for the 2 cards combined.

At this point you cannot add a third card with the first two half and half, however small the overlap, unless the bottom card is completely on the table. So now play around, try assembling a number of cards with small overlaps, put these on table so bottom card only slightly sticks out. See how much you can overhang 1 or 2 sat on this. It's not long before you can place a card that is in effect completely beyond the edge of the table.

Cards higher up can be overhung more than those below – because they have less weight acting on them. It is this fact which allows the gap to be closed with something approaching a hemisphere. Unless you think about it this might seem at first seem counter intuitive to many.



This is how you form a dome like roof; if you don't overhang the higher stones very much the dome becomes more pointed. In some instances (as we shall see with the Kielder 'Wave Chamber' in the next issue) the pointed-ness is likely to be an unwitting side effect of not realising how much faster you need to overhang the top stones to form the dome and a lack of confidence/experience. Before we look a bit more deeply into exactly how this works in theory try thinking of a solid circle drawn on a pixelated screen. At the sides of the circles the steps in are relatively few compared to height, towards the top/bottom, the steps in are relatively great compared to height.

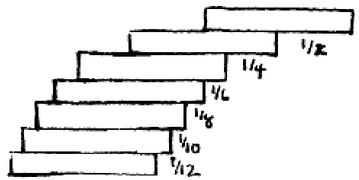
There is a nice mathematical equation associated with how much you can overhang things placed on top of each other (provided they are of a uniform size and shape). I know many eyes are now glazing over, but it is relatively simple, but if you must just skip the next three paragraphs (and don't forget to wear a hard hat if you ever try corbelling).

So here we go…
$^1/_2+^1/_4+^1/_6+^1/_8+^1/_{10}$. Hopefully you can see a pattern, if you have 1 card the denominator (bottom part of the fraction) is 2, 2 cards its 4, 3 cards its 6 ad infinitum, basically the denominator steps up in multiples double the number of cards. The mathematical amongst you will have noticed that the full formula is $^1/_2+^1/_4+^1/_6+^1/_8+\cdots+^1/_{2n}$

In terms of getting a card to be wholly beyond the table you should be able to achieve this in about 4 cards, five if you're playing safe. The astute amongst you will however realise that because additional cards require smaller overlaps at the bottom you begin to suffer from very diminishing returns. Theoretically to progress another half a card out would require another 7 cards (11 total) and to overlap by two in total would require another 20 (31 total).

This paragraph is purely aimed at those with more than a passing curiosity in mathematics… The formula above can also be written as $½H_n$ where H_n is the n^{th} harmonic number and $H_n=1+^1/_2+^1/_3+^1/_4+\cdots+^1/_n$ which is also $\sum_{k=1}^{n} {^1/_k}$

From this you an calculate the maximum overhang for n boxes as $½H_n$.



I should point out that I leant heavily on *"Professor Stewarts Hoard of mathematical Treasures"*, I Stewart, Profile Books London, 2009, in preparing this analysis.

You will also hopefully have noticed that the equation is in fact upside down essentially what it means is that for each card you add you need to move the first card a little further onto the table. The most you can overhang the top card is always ½ but you have in effect moved the centre of gravity out beyond the edge so the whole pile needs to be moved in a little. This would be difficult to apply to dry stone walling because stones vary but not impossible as long as you know how many layers (or thereabouts) you intend to have and how high you are going to go.

Problems occur where thickness (and length but that's another issue) varies – then calculating relative overlaps would be a nightmare.

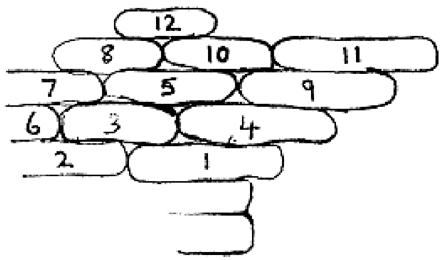
The end result of this is a smooth (ish) curve which can be represented by a graph. I haven't worked out a suitable way of reproducing this here but if you have a look at http://en.wikipedia.org/wiki/Harmonic_number and turn your head sideways you'll get the idea. Note how the steps increase as the parabola flattens (i.e. the roof).

The importance of this idea – apart from having some fun messing about with playing cards - is that set up mathematically the corbelling is stable under its own weight. In reality you are likely to reduce the size of the steps to ensure stability. If you play around with the cards you soon discover that stacks with large overhangs are inherently unstable and difficult to add to. If you form a 'bridge' (say by creating two opposing overhangs off of upturned mugs with a lintel closing the gap – yes it is both sad and amazing what I get up to in my spare time albeit with index cards because the playing cards are buried in a box somewhere) you should be able to check its stability by placing additional cards on the lintel. If the steps are near the stability limit then it will not take much before the whole collapses because at the limit the structure is only stable under its own weight and no more – unlike a true arch.

In reality your stones are not all nice and uniform so all that I have discussed so far is just idealised theory. As with all things dry stone you do not need to understand the theory to get it to work in practice, trial and error and intuitive skill hold many in good stead. However if you understand the theory, or at least have an inkling of an understanding you are less likely to make the mistakes that lead to problems further down the line – in terms of the actual building of the corbelled structure now, or what happens to it 20 or 30 years down the line.

The most obvious variation to the freestanding self supporting card 'model, is that you can counterbalance the corbel by placing another stone on its tail. This means the corbels can be overhung more. In practice you would build this way even if you are only stepping the stones out slightly because you are adding to the stability. This is the principle of cantilevering.

The diagram shows a stylised order for setting cantilevered stone. Note that stone 9 is likely to be displaced when 11 is sat on it because 10 barely overlaps. The chances of this happening are reduced by placing 12 before 11 as this reduces the potential for 10 to be levered up by 9. Generally the more you can overlap the cantilevering stone the more the better the corbel will be held, although the compromise is that the next corbel will not be as long or secure in its own right.



This has all sorts of implications… back to the playing cards. Or in my case empty DVD boxes as index cards are I think too thin/flexible. Overhang the bottom card about half way; lap another card onto the tail of it on the table, overlapping by about a quarter. Now you can place another card on the overhang without the whole lot toppling. You can take all sorts of liberties by increasing the weight (i.e. stacking cards) on the tail. Gaps can be closed relatively quickly. The problem comes when you remove some of the counterweight (or they move as is the potential in a structure over time) then the whole lot suffers a catastrophic failure. (Stones are likely to be more stable than cards or DVDs which are relatively smooth, in reality friction will mean stones can tip a little more before the whole fails. It is however still a 'slippery slope' in terms of overall stability.) The corbelling is no longer stable under its own weight. There can be a temptation to extend the stones further than the ideal, especially when trying to close the gap quickly. You must resist!

Next time we'll have a look at corbelling more from a practical standpoint, that of course does not mean you'll be so lucky as to get away without any theory.

Sean Adcock